IMPORTANT

THE **MATH COMPETENCY STUDY GUIDE** IS INTENDED TO BE AN <u>AID FOR THE REVIEW</u> OF ALGEBRA SKILLS. IT IS NOT INTENDED TO DEVELOP ALGEBRA SKILLS FOR THOSE WHO HAVE NOT HAD ALGEBRA IN THE LAST 5 YEARS.

THE ALGEBRA STUDY GUIDE AND SAMPLE ALGEBRA TEST CONSIST OF EXAMPLES AND PROBLEMS INVOLVING: REAL NUMBERS, ALGEBRAIC EXPRESSIONS AND SOLUTIONS OF EQUATIONS.

The following examples in the study guide involve REAL NUMBERS:
    1, 2, 3, 4, 5, 26, 27, 28, 29, 33.

The following problems in the sample test involve REAL NUMBERS:
    1, 2, 3, 4, 5, 27, 28, 30.

The following examples in the study guide involve ALGEBRAIC EXPRESSIONS:
    6, 7, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 30.

The following examples in the sample test involve ALGEBRAIC EXPRESSIONS:
    6, 7, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 26, 29, 33.

The following examples in the study guide involve SOLUTIONS OF EQUATIONS:
    8, 9, 10, 22, 23, 24, 25, 31, 32.

The following examples in the sample test involve SOLUTIONS OF EQUATIONS:
    8, 9, 10, 22, 23, 24, 25, 31, 32.

ALGEBRA STUDY GUIDE

Example 1

Find $6 - (-2)$
    To subtract, change the sign of the second number and add.
    $6 - (-2) = 6 + 2 = 8$

Example 2

Evaluate $\dfrac{5m + n}{m - n}$ if $m = -4$ and $n = 2$.
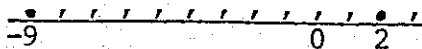   Replace m with -4 and n with 2.
    $\dfrac{5(-4) + 2}{-4 - 2} = \dfrac{-20 + 2}{-4 + (-2)} = \dfrac{-18}{-6} = 3$

Example 3

Choose the smaller number: $-9, 2$
    The smaller number is to the left on a number line.
    Place -9 and 2 on a number line.
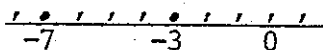

    Since -9 is farther to the left it is smaller.

Example 4

Choose the smaller number: $-3, -7$
    Use a number line.

    -7 is smaller since it is further to the left.

Example 5

Choose the smaller number: $|2|, |-7|$
    $|x|$ means the absolute value of x. Since $|x|$ is always non-negative, $|2| = 2$ and $|-7| = 7$.
    The $|2|$ is smaller.

SIMPLIFY:

Example 6

$8x - 3x + x$
    Use the distributive property, and the fact that $x = 1x$.
    $8x - 3x + x = (8 - 3 + 1)x = 6x$

Example 7

$2(3m - 5) - (2 - m)$
    Use the distributive property.
    $2(3m - 5) - (2 - m) = 2(3m - 5) - 1(2 - m)$
    $= 6m - 10 - 2 + m$
    $= 7m - 12$

SOLVE EACH EQUATION:

Example 8

$3p - 7 = 23$
    Add 7 to both sides. $3p - 7 + (7) = 23 + (7)$
            $3p = 30$
    Multiply both sides by $\frac{1}{3}$. $\frac{1}{3} \cdot 3p = \frac{1}{3} \cdot 30$
            $p = 10$

/

SOLVE EACH EQUATION:

Example 9

$$2p - (7p + 3) = 2$$

Simplify: $2p - 7p - 3 = 2$

$$-5p - 3 = 2$$

Add 3 to both sides:

$$-5p - 3 + (3) = 2 + (3)$$
$$-5p = 5$$

Multiply both sides by $-\frac{1}{5}$

$$-\frac{1}{5}(-5p) = -\frac{1}{5}(5)$$
$$p = -1$$

Example 10

$$-6m + 2 < -10$$

Add -2 on both sides. $-6m + 2 + (-2) < -10 + (-2)$

$$-6m < -12$$

Multiply both sides by $-\frac{1}{6}$. Recall that when multiplying both sides of an inequality by a negative number, the direction of the inequality symbol must be turned around.

$$\frac{1}{6}(-6m) > -\frac{1}{6}(-12)$$
$$m > 2$$

Example 11

Multiply: $8p(p^2 + 7p + 2)$

Use the distributive property.

$$= 8p(p^2) + 8p(7p) + 8p(2)$$
$$= 8p^3 + 56p^2 + 16p$$

Example 12

Add: $(9y^3 + 2y^2 + 3y + 2) + (8y^3 + 5y - 7)$

Add only like terms (those with the same exponents).

$$(9y^3 + 8y^3) + (2y^2) + (3y + 5y) + (2 - 7)$$
$$= 17y^3 + 2y^2 + 8y - 5$$

Example 13

Multiply: $(8k - 1)(2k + 3)$

Multiply the first terms. $(8k)(2k) = 16k^2$
Multiply the outside terms. $(8k)(3) = 24k$
Multiply the inside terms. $(-1)(2k) = -2k$
Multiply the last terms. $(-1)(3) = -3$

Add them together. $16k^2 + 24k + (-2k) + (-3)$

$$= 16k^2 + 22k - 3$$

Example 14

Multiply: $(3m - 5)^2$

Square the first term. $(3m)^2$
Take twice the product of the two terms. $2(3m)(-5)$
Square the last term. $(-5)^2$

Add them together. $(3m)^2 + 2(3m)(-5) + (-5)^2$

$$= 9m^2 - 30m + 25$$

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Example 15   Divide: $x^3 - 6x^2 + 16x - 15$ by $x - 2$

Work as follows:

$$\begin{array}{r} x^2 - 4x + 8 \\ x-2 \enclose{longdiv}{x^3 - 6x^2 + 16x - 15} \\ \underline{x^3 - 2x^2} \\ -4x^2 + 16x \\ \underline{-4x^2 + 8x} \\ 8x - 15 \\ \underline{8x - 16} \\ 1 \end{array}$$

Answer: $x^2 - 4x + 8 + \dfrac{1}{x - 2}$

FACTOR

Example 16 $16m^2 - 25$

Use the difference of two squares, $x^2 - y^2 = (x + y)(x - y)$

$16m^2 - 25 = (4m)^2 - (5)^2 = (4m + 5)(4m - 5)$

Example 17 $x^2 + 12x + 20$

Since the coefficient of the x^2 term is 1, find 2 numbers whose product is 20 and whose sum is 12.

     $20 \times 1 = 20$ $20 + 1 = 21$ no

     $10 \times 2 = 20$ $10 + 2 = 12$ yes

Therefore:

     $x^2 + 12x + 20 = (x + 10)(x + 2)$

Example 18 $2p^2 - p - 21$

Use trial and error

     $(2p - 3)(p + 7) = 2p^2 + 11p - 21$ wrong

     $(2p + 7)(p - 3) = 2p^2 + p - 21$ wrong

     $(2p - 7)(p + 3) = 2p^2 - p - 21$ correct

Example 19   Multiply: $\dfrac{9m^2}{2(m - 4)} \cdot \dfrac{8m - 32}{6m}$

Factor and then cancel $(m - 4)$ & m on top and bottom.

$$\frac{9m^{\cancel{2}}}{2\cancel{(m - 4)}} \cdot \frac{8\cancel{(m - 4)}}{\cancel{6m}} = \frac{9m}{2} \cdot \frac{8}{6} = \frac{72m}{12} = 6m$$

Example 20   Divide: $\dfrac{6r^2}{r^2 + 2r - 15} \div \dfrac{3r}{r^2 - r - 6}$

Factor. Invert the second fraction and multiply.

$$\frac{\overset{2}{\cancel{6}}r^{\cancel{2}}}{(r + 5)\cancel{(r - 3)}} \cdot \frac{\cancel{(r - 3)}(r + 2)}{\cancel{3r}} = \frac{2r(r + 2)}{r + 5}$$

Example 21   Write as a single fraction: $\dfrac{4}{m} + \dfrac{3}{2m} - \dfrac{5}{7m}$

The least common denominator is 14m.

$$\frac{4}{m} + \frac{3}{2m} - \frac{5}{7m} = \frac{14\cdot 4}{14\cdot m} + \frac{7\cdot 3}{7\cdot 2m} - \frac{2\cdot 5}{2\cdot 7m}$$

$$= \frac{56}{14m} + \frac{21}{14m} - \frac{10}{14m} = \frac{56 + 21 - 10}{14m} = \frac{67}{14m}$$

Example 22   Solve: $x^2 - 3x = 10$

Set the equation equal to zero: $x^2 - 3x - 10 = 0$

Factor: $(x - 5)(x + 2) = 0$

Set each factor equal to zero: $x - 5 = 0$ $x + 2 = 0$

$x = 5$ $x = -2$

The two solutions are 5 and -2

Example 23   Solve: $\dfrac{6}{m} + \dfrac{8}{m} = 1$

Multiply through by m.

$$m\cdot\frac{6}{m} + m\cdot\frac{8}{m} = m\cdot 1$$

$$6 + 8 = m$$

$$14 = m$$

Example 24   Solve the system: $2x - 3y = -8$

$3x + 5y = 7$

Multiply the top equation by -3 and the bottom equation by 2.

$-6x + 9y = 24$

$\underline{6x + 10y = 14}$

Add. $19y = 38$

$y = 2$

To find x, replace y with 2 in the first equation.

$2x - 3(2) = -8$

$2x - 6 = -8$

$2x = -2$

$x = -1$

The solution is the ordered pair $(-1, 2)$

Example 25   Solve: $p = \dfrac{3k}{7r}$ for r.

Multiply both sides by 7r: $7r\cdot p = 7r\cdot\dfrac{3k}{7r}$

$7pr = 3k$

Multiply by $\dfrac{1}{7p}$: $\dfrac{1}{7p}\cdot 7pr = \dfrac{1}{7p}\cdot 3k$

$r = \dfrac{3k}{7p}$

Example 26   The square roots of 144 are 12 and -12, since $12^2 = 144$ and $(-12)^2 = 144$. Also the roots of $144m^8$ are $12m^4$ and $-12m^4$.

Example 27   Multiply: $\sqrt{7}\cdot 5\sqrt{3}$

$\sqrt{a}\cdot\sqrt{b} = \sqrt{ab}$. Therefore, we get $5\sqrt{7\cdot 3} = 5\sqrt{21}$

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Example 28   Simplify: $-7\sqrt{32} + 8\sqrt{50}$

To simplify $\sqrt{32}$, work as follows: $\sqrt{32} = \sqrt{16 \cdot 2}$
$= \sqrt{16} \cdot \sqrt{2} = 4\sqrt{2}$
Also, $\sqrt{50} = \sqrt{25 \cdot 2} = \sqrt{25} \cdot \sqrt{2} = 5\sqrt{2}$
$-7\sqrt{32} + 8\sqrt{50} = -7(4\sqrt{2}) + 8(5\sqrt{2})$
$= -28\sqrt{2} + 40\sqrt{2}$
$= 12\sqrt{2}$

Example 29   Solve: $3^2 \cdot 3^4$

Since $a^m \cdot a^n = a^{m+n}$

$3^2 \cdot 3^4 = 3^{2+4} = 3^6$

Example 30   Solve: $(x^2)^3$

Since $(a^n)^m = a^{n \cdot m}$

$(x^2)^3 = x^{2 \cdot 3} = x^6$

Example 31   The total cost of 3 items is $40.00. If the first item costs $15.00 and the second item costs $18.00, how much does the third item cost?

Solution:

The equation is: $x + y + z = 40$

Since the cost of two items is known, we can substitute those values in the equation.

$15 + 18 + z = 40$ Now solve for z

$33 + z = 40$ Add $15 + 18$

$z = 40 - 33$ Subtract 33 from 40 to solve for z

$z = \$7.00$

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Example 32   If 2/3 of a number is 30, what is the number?

Solution:

The equation is: $(2/3)x = 30$ Now solve for x

$x = 30 \cdot (3/2)$ Multiply 3/2 by 30

$x = 15 \cdot 3$ Cancel the 2 on the bottom
                    and reduce 30 to 15 on top

$x = 45$

Example 33   Simplify: $\sqrt[4]{48}$

$\sqrt[4]{48} = \sqrt[4]{2^4 \cdot 3}$ 48 is equivalent to $2^4 \cdot 3$

Since $\sqrt[n]{a^n} = a$ and $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$

$\sqrt[4]{48} = \sqrt[4]{2^4 \cdot 3} = \sqrt[4]{2^4} \cdot \sqrt[4]{3} = 2\sqrt[4]{3}$

SAMPLE ALGEBRA TEST

Complete the following.

1. $-5 - 9$

2. If $a = 3$ and $d = -2$

$$\frac{2ad}{a - d} =$$

Choose the smaller number.

3. $-3, 1$ 4. $-2, -5$ 5. $|-2|, |-5|$

Simplify.

6. $-2x + 5x - 9x$

7. $4(2x + 1) - (x - 2)$

Solve.

8. $4x - 2 = 10$

9. $6y - (3y + 4) = y$

10. $-2x + 6 < 4$

11. Multiply: $5mn(3m^4n + 2m^3)$

1. _____

2. _____

3. _____

4. _____

5. _____

6. _____

7. _____

8. _____

9. _____

10. _____

11. _____

7

12. Add: $(x - 2) + (3x^2 + 2x - 4)$

12. _____

13. Multiply: $(2x + 3)(x - 5)$

13. _____

14. $(2x + 3y)^2$

14. _____

15. Divide: $x^3 + 4x^2 + 7x + 12$ by $x + 3$

15. _____

Factor completely.

16. $s^2 - t^2$ 17. $x^2 + 6x + 8$

16. _____

17. _____

18. $5r^2 - 13r + 6$

18. _____

19. Multiply: $\dfrac{3x^2}{x + y} \cdot \dfrac{2(x + y)}{6}$

19. _____

20. Divide: $\dfrac{6a^2b^2}{a^2 - b^2} \div \dfrac{3a^2b^2}{a^2 - 2ab + b^2}$

20. _____

21. Write as a single fraction: $\dfrac{1}{x} + \dfrac{3}{y} - \dfrac{x}{xy}$

21. _____

SOLVE:

22. $x^2 - 5x - 14 = 0$

22. _____

23. $\dfrac{2}{x} + \dfrac{3}{x} = 10$

23. _____

24. $x - 4y = -8$
    $x + 2y = 10$

24. _____

25. $R = \dfrac{K}{mD}$ for D

25. _____

26. What are the square roots of $16b^8$?

26. _____

27. Multiply: $\sqrt{5} \cdot 8\sqrt{3}$

27. _____

28. Simplify: $3\sqrt{20} - 2\sqrt{5}$

28. _____

29. $(3a^3b^3)^2 =$

29. _____

30. $2^3 + 3^0 =$

30. _____

31. If $a = 11$ and $b = 4$, what is the value of c
    if the sum of a, b and c is 25?

31. _____

32. If 4/5 of a number is 24, what is the number?

32. _____

33. $\sqrt[4]{32x^4y^4} =$

33. _____

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Sample Algebra Test Answers

1. -14

2. $-\dfrac{12}{5}$

3. -3

4. -5

5. $|-2|$

6. $-6x$

7. $7x + 6$

8. 3

9. 2

10. $x > 1$

11. $15m^5n^2 + 10m^4n$

12. $3x^2 + 3x - 6$

13. $2x^2 - 7x - 15$

14. $4x^2 + 12xy + 9y^2$

15. $x^2 + x + 4$

16. $(s + t)(s - t)$

17. $(x + 2)(x + 4)$

18. $(5r - 3)(r - 2)$

19. x^2

20. $\dfrac{2(a - b)}{a + b}$

21. $\dfrac{y + 2x}{xy}$

22. $\{-2, 7\}$

23. $1/2$

24. $4, 3$

25. $D = \dfrac{K}{Rm}$

26. $\pm 4b^4$

27. $8\sqrt{15}$

28. $4\sqrt{5}$

29. $9a^6b^6$

30. 9

31. 10

32. 30

33. $2\sqrt[4]{2}\ xy$